# Memristics: Memory is more than Storage

## *Memristive memory is able to surpass the conceptual limits of computational storage methods*

**Rudolf Kaehr Dr.phil.**



### Abstract
The relationship between memory and computation was not always a happy one. Once fixed by John von Neumann's conceptualization of the practice of engineering solutions for practical computer architectures, it has become the ultimate paradigm of architecture but ending now into its permanent bottleneck foreclosing the former interactivity of memory and computation. Insights into mnemonics of the Ancient solutions to the process of memorization are slowly recovering from the military hierarchy of commander and commanded, and the reduction of memory to storage. Memristive systems are prepared to re-dynamize the interplay of memory and computation again. Some orientation towards conceptual generalizations of memristive approaches is given with the use of poly-categorical methods.


## 1. "Memory without Record" (Heinz von Foerster)

### Memristors as Logic

*"The biggest new news about memristors, though, came in a paper in Nature last week, in which HP announced that the devices can also perform logic functions. In other words, Wiliams said, a memristor can act as both a storage element and a logic element, or* "a lock as well as a gate.*"*

*"*There's nothing else I'm aware of that performs both of those functions simultaneously,*" he said."*

*Williams said there is an* "intriguing possibility" *that if you could use the same structure to do actual computing as well as storage, you*

*could send the program to where the data is and execute the problem where the data is stored. Of course, that all depends on what the performance of memristor-based devices ends up being, compared with traditional CPUs and memory systems.*
http://blogs.pcmag.com/miller/2010/04/memristors_a_flash_memory_comp.php#more

**Memory is more than storage**

*"Note the double closure of the system which now recursively operates not only on what it "sees" but on its operators as well."* (Heinz von Foerster, On constructing a Reality, in: Observing Systems, p. 305, 1984)

Storage implementation by flip-flops based on NAND or NOR gates are first-order concepts realizing storage and computation with the help of an "external" timer.
Memristive realizations are of second-order, they are not genuinely implemented by NAND-derivatives build by IMP but by a new kind of second-order construction. Because of their second-order status they are not primarily emulating storage but memory. Memory, in this generalized sense, is a self-referential construct, allowing to change the memorized object while memorizing, hence the object is not simply stored as a record, but is accessible to re-interpretation.

This observation goes beyond the famous *"stateful logic"*-concept (Williams), re-unionizing memory and logic, because the formulation *"memory and logic"* is first re-institutionalizing the old concepts of logic, i.e. the binary 2- or many-valued logics for digital or analog computation, and memory as an a-temporal, non-directional and non-reinterpretable way of storage albeit in a new way of a close interaction between both omitting loss by resource-consuming data-transfers between both, logic and memory.

Therefore, Chua's statement that memristors are synapses has to be reframed and reconsidered as a second-order statement.

Hence, the highly abstract conceptual interpretation of memristive flip-flops as sketched in my *"Memristic flip-flop"* -paper gets some crucial concretization towards the difference of the first-order concept "*storage*" based on NAND flip-flops, or equally implicational implementations, and the second-order memristive “chiasm” (flip-flop) based on distributed memristive (IMP, F)-constructions.

> *"What has to be saved by the memristive device are in fact not the first-order data of FF but the conditions of the possibility of the data, i.e. the ‘data’ of the matching condition of the composition of the master and the slave flip-flop morphisms (processes). From those conditions, as second-order constructs, the first-order data might be reconstructed on the base of the second-order data saved by the memristive device.*
>
> *In other words, the history saved by the memristor are not the primary data but the data of the history. Historical data are data of data. Those second-order data might then be used to continue processing on the first-order level of the flip-flop.*
>
> *As a metaphor, the data of an observer of a data processing system are not the data of the observed system. But such observer-depending data of second-order might be given ‘back’ to the observed, i.e. first-order system to continue its game. Hence, the memristor is playing the game of an observer which is lending or giving away his data to the observed system.*
> *The memristive system is primarily storing the rules of the observed game and only secondarily*
> *the data involved.”*
>
> *HP’s construction is using the flip-flop channel as a memristive buffer, and is not yet exploiting the possibilities of the interactivity of the master/slave-relationship by the involvement of the memristor. The concepts of complementarity, simultaneity and antidromicity are not yet used in the construction of HP’s memristive FF device.”*
> (Flip-Flop, p.17)

Hence, my first critical analysis, gets concretized by the introduction of the difference between *first*-order “storage” and *second*-order “memory".

Memories are interpreted data. Data without interpretations are records. Data cannot be changed as data, memories are changed by contexts, i.e. by the change of their operators. Memories without contexts don’t exist; for stored records contexts don’t exist.

How are second-order memories, based on memristors, working?

## 2. Second-order flip-flops

A transition from first-order flip-flops to second-order flip-flops is not mixed with the strategy to enlarge the range of decisions from bi-stable to tri-stable or multi-stable devices of first-order. This goes together with the statement that a transition from mono-contextural to polycontextural logics is not in any complicity with the change from 2-valued to ternary and multiple-valued logics. Such enlargements or precision of logical parameters don't surpass the limits of mono-contextural logics and concept developments.

Interpretation of data is computing data. Hence, memories are data of data; and therefore a unit of data and computation (logic).

If data have to be computed, before they eventually could be stored, second-order memories don't have records.

A closer look at the structure of computed, i.e. constructed data for non-storage memories, would have to go into the theory of "*Eigenbehavior*", the cybernetician von Foerster introduced to understand the *"operational closure of cognitive systems".* In other words, the study of computation and memory for memristive systems should be connected with the results of the studies towards a theory of *living systems* as it was developed by the approaches of second-order cybernetics. Because of Chua's interpretation of synapses as memristors, such a connection with cybernetics could be of mutual productivity.

The whole interplay of memory and computation might be radically reduced to an operational interplay of operators and operands in a contexture. With operators representing computation and operands representing memory as stored data.

- first-order category theory (CAT): objects, morphisms, composition and yuxtaposition
- second-order category theory: dissemination of first-order CAT and new rules between CATs.

It could be said, that von Foersters insistence on recursivity and self-referentiality is based on the fact that there is no memory-function explicitly involved in his mathematical modeling strategies. Self-referentiality, say as self-repair, is thought by von Foerster as “repair of repair”, i.e by an iteration/accretion of the term. And obviously, but not necessarily, the first and the second term have to be at least similar if not identical. A concept of self-x as ” *x of x”* is not working with a difference in the terms, say *“x of y”*, like self-repair as repair of selling, instead, repair of repair.

In contrast, Gunther’s philosophical “proemial" (chiastic) modeling was fundamentally involved with memory-functions, albeit this was not always explicitly stated as such, and his formalism is not operative enough to be seriously applied in the construction of second-order devices.

On the other hand, recursive functions and contemplations on it are still more descriptive than operative. For recursive function the address of re-entry in a circular calculation is pre-defined, and the question, how can a function miss its re-entry-address, appears to be strictly absurd.

For proemial operativity, the interchange of operator and operand has to be installed with the help of a ‘*coincidence*’ relations, otherwise the process of self-application might be confused and, because of the complexity of the construction, might easily miss the intended re-entry-point for another destination (slot). Therefore, the concepts of proemial relationship got a concretization as a chiasm with the introduction of the coincidence or similarity relations. A further formalization of proemiality was introduced with the construction of poly-category-theoretical diamonds.

It seems to be reasonable to interpret and implement such constituents like ‘*coincidence*’ relations in chiasms as memory functions.

## *Are memristors non-trivial machines?*

### Non-Trivial Machines

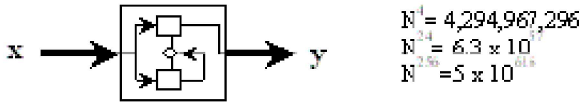
Non-trivial machines had been introduced by Heinz von Foerster.

Non-trivial machines have the following properties:

(i) Synthetically determined;
(ii) History dependent;
(iii) Analytically indeterminable;
(iv) Unpredictable.

Driving function: y = F(x, y)
State function: z’ = Z (x, z)
Z = internal state.

$N^4$= 4,294,967,296




**Procedures**

(i) *Read* the input symbol x.
(ii) *Compare* x with z, the internal state of the machine.
(iii) *Write* the appropriate output symbol y.
(iv) *Change* the internal state z to the new state z’.
(v) *Repeat* the above sequence with a new input state x'.

Trivial machines have additionally to their history-independence a “predictable” behavior, non-trivial machines behave “unpredictable”. Is this true for memristors? If yes, in which sense is the behavior of memristors unpredictable?
The behaviors of memristors is interpretable in two distinct ways: 1. as digital, and 2. as analog.

**Unpredictability**

*"Ionized atomic* degrees of freedom *define the internal state of the device.*‘’

**History dependence**

*"New possibilities in the understanding of neural processes using memristive memory devices whose response depends on the* whole dynamical history *of the system.”* (Kim, New Scientist, 2009)

Memristors are in fact “*assemblies of nanoparticles*" (Kim, 2009), therefore, their behavior is not “analytically” pre-defined, their behavior, depending on the context of the system, has to be interpreted. This might happen as an abstraction of identity, producing a predictable binarity of values, or it might be interpreted analogously, producing a non-predictable set of values.

History-dependence is not yet a criteria for memristive behavior if the kind of dependency is not properly characterized. Classical finite state machines are behaving “history-dependent” too. The dependence on the internal state of a machine and its history of events are typical for finite state machines albeit they are not time-dependent.

What is the difference to memristic systems?
The internal states of a finite state machine are first-order states. They are states of the machines and not states of states of the machine. Without this difference, memristic systems collapse to ordinary finite state machines, and the proclaimed paradigm change is lost.

Obviously, memristors are not representing trivial machines because their *“resistance depends on the internal state of the system”*, while trivial machines, mono-contextural as well as ‘polycontextural’, are *“history independent”.* Also memristors have a strict distinction between input and output functionality, their out-put is depending not only on the input and the definition of the function but also on the history of the former input/output relation. Hence they are *“history dependent”*.

Are memristors therefore non-trivial?

Because functionally complete logical functor-sets are representing trivial machines, the characterization of memristors as non-trivial machines might be in conflict with the understanding of memristors as *material implications* together with the functional completeness of a logic with implication and a negative constant, {IMP, F}. Complete junctional sets in logic theories are decidable, hence, their behavior is predictable.
This characterization by logically complete sets is only halve the story of the possible behaviors of memristors.

**Further Notes**
A finite state machine has a state but not a memory of a state.
A memristive machine has a state of a state, i.e. a *meta-state* as a memory, therefore a memristic machine is not a finite state machine.
A meta-state always can be taken as a simple state because a reduction from an as-abstraction to an is-abstraction is directly possible because the necessary informations are stored in the meta-state. From *“x as y is z”* there is an easy way to reduce it to *“x is x*”.
A memristive machine, then, is a machine with a tensed time, while finite state machines are not tensed machines. Their temporality is of first-order, memristic time is of second-order, i.e. an interpretation of a state of a state.
Todays interpretation of memristors as memory devices in an ANN is reducing the possibility of second-order learning to simple first-order

learning as trained adaption.

Further analysis will be published as *“Towards Abstract Memristic Machines".*

# 3. Are memristors morphograms?

## 3.1. Physical model

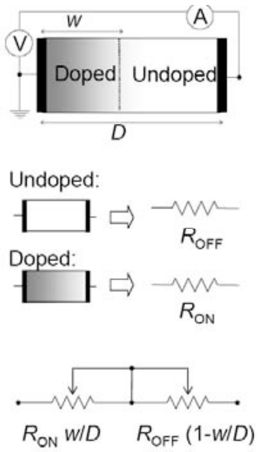
### 3.1.1. Physical realization

The study of the behavior of memristors has been focussed on the abstract and non-located functionality of memristors.
The definition of memristors, physically as doped/undoped nanodevices, and logically as material implication with negativity, had been restricted to a context-free, i.e. non-located design and understanding.

But it seems, that memristors, in contrast to classical logical devises, are not necessarily connected to a single physical or logical place to develop their functionality.

"x is the width of the doped region, referenced to the total length D of the TiO2 layer, and $R_{off}$ and $R_{on}$ are the limit values of the memristor resistance for w = 0 and w = D. The ratio of the two resistances is usually given as 102 - 103.”

**Phenomenological Description**
Strukov, Stewart, Snider & Williams, Nature 453 80 (2008)




Ionic drift:

$$\frac{dw(t)}{dt} = \mu_V \frac{R_{ON}}{D} i(t)$$

Electronic current:

$$v(t) = \underbrace{\left[ R_{ON} \frac{w(t)}{D} + R_{OFF}\left(1 - \frac{w(t)}{D}\right)\right]}i(t)$$

$$M(q) = R_{OFF}\left[1 - \frac{\mu_V}{D^2} R_{ON} q(t)\right]$$

**Formula**

$$R_{mem}(x) = R_{on}\, x + R_{off}(1 - x) = R_{off} - (R_{off} - R_{on})\, x\,,$$

$$x = \frac{w}{D} \in (0,\ 1).$$

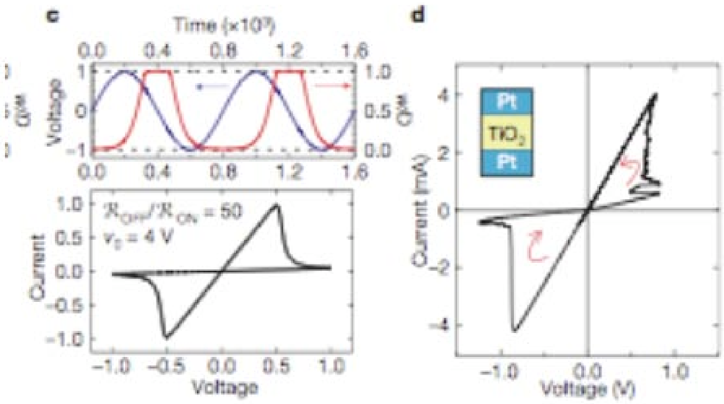
*"x is the width of the doped region, referenced to the total length D of the TiO2 layer, and $R_{on}$ and $R_{off}$ are the limit values of the memristor resistance for w = 0 and w = D. The ratio of the two resistances is usually given as 102 - 103."*

### 3.1.2. Physical diagrams, the pinged-hysteresis loop

What is the category-theoretical modeling for mathematicians is the data-representation of the behavior of memristors under specific physical conditions by curve-diagrams.
It is stated that such a modeling of a data-function with a purely structural method like polycontextural monoidal categories is opening up not only a conceptual insight into the mechanism of the pinched-hysteresis loop but also interesting generalizations in a structural and domain-specific sense.

It seems that such an approach had its justification in the historical example of the understanding of physical events as an emulation of the abstract figuration of Chua's definition of a memristor.

> *Medha Haridas et al*
> "That's when Greg Snider, who had worked with Kuekes on the Teramac, brought the Chua memristor paper from the September 1971 IEEE Transactions on Circuits Theory. Chua's paper had included a *graph* that looked suspiciously *similar* to the *experimental* data that were being collected.
> The graph described the current-voltage (I - V) characteristics that Chua had plotted for his memristor. Chua had called them *"pinched-hysteresis loops"*, I - V characteristics for the platinum device were called *"bow ties.*"
> A pinched hysteresis loop looks like a diagonal infinity symbol with the center at the zero axis, when plotted on a graph of current against voltage. The voltage is first increased from zero to a positive maximum value, then decreased to a minimum negative value and finally returned to zero. The bow ties were nearly identical. That's not all."
> http://www.ijcee.org/papers/195-E568.pdf




time. In all cases, hard switching occurs when $w/D$ closely approaches the boundaries at zero and one (dashed), and the qualitatively different $i$–$v$ hysteresis shapes are due to the specific dependence of $w/D$ on the electric field near the boundaries. **d**, For comparison, we present an experimental $i$–$v$ plot of a Pt–$TiO_{2-x}$–Pt device[21].

**Translation (diagram above)**

Possible translation of the pinched –

hysteresis loop for voltage and current into a composition of morphisms :

**left –loop** : $h_1 : f_1 \circ g_1 : f_1 \longrightarrow g_1$

corresponds :

$([0.0,\ 0] \longrightarrow [1.0,\ 0]) \circ ([1.0,\ 0] \longrightarrow [1.0,\ 4\,]) \Longrightarrow ([1.0,\ 4] \longrightarrow [0.0,\ 0])$,

**right –loop** : $h_2 : f_2 \circ g_2 : f_2 \longrightarrow g_2$

corresponds : $([0,\ 0.0\,] \longrightarrow [0,\ -1.0]) \circ$

$([0,\ -1.0] \longrightarrow [-4.1,\ -1.0]) \Longrightarrow ([-4.1,\ -1] \longrightarrow [-0,\ 0.0])$

**pinch** : $h_1 \circ h_2 : (f_1 \longrightarrow g_1) \circ\ (f_2 \longrightarrow g_2)\ |$

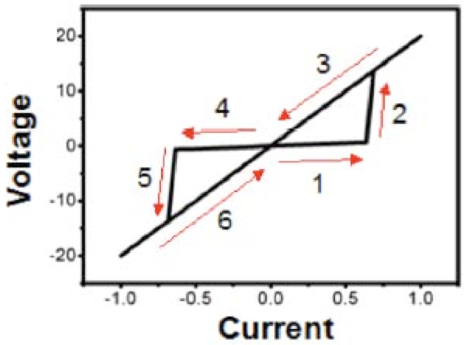
corresponds : full hysteresis loop with pinch $([-0,\ 0.0],\ [0,\ 0.0])$.

### Kim’s simplified diagram

It seems to be only an easy step to accept a category-theoretic formualtion considering Kim's simplification of the data-loop to a directed arrow diagram.

$\text{left} - \text{loop}_1 \ = (1, 2, 3),$
$\text{right} - \text{loop}_2 = (4, 5, 6),$
$\text{pinch} = \text{loop}_1 \cap \text{loop}_2,$
$\text{h-loop} = \text{loop}_1 \cup \text{loop}_2.$

This step to conceptual modeling is not yet concerning a categorization of the *mathematical* formulas describing the physical behaviors.




### 3.1.3. Pinched-hysteresis loops as interchangeability

As it was shown in previous papers, there is a close similarity between the conceptual structure of the definition of a memristor and diamond category-theoretical constructions of chiasms and interchangeability.
Hence, it doesn't comes as a surprise to see a chance to model the *data-flow* schemes of memristors with the help of diamond categories. It gives probably some more insight into the structure of the *temporal* behavior of memristors if this behavior is modeled as an *interchangeability* of its constituents, which are "positive maximum", "negative minimum" and "positive zero", "negative zero" of voltage (V) and current (mA) as a *"bow tie"* movement.

With $(f_1 \circ g_1)$ for the (f, g)-*left* loop-part, $(g_2 \circ f_2)$ for the (f, g)-*right* loop-part of the distributed double loop with "zero crossing property" the pinched-hysteresis loop gets a structural model. The involved domains $\mathcal{U}_{1.1}$ and $\mathcal{U}_{1.2}$ here are the *positive* and the *negative* value-space of voltage (V) and current (mA), hence might be modeled as belonging to a common domain $\mathcal{U}_1$.

**Cross – interchangeability of two internal memristive domains**

$$\mathcal{U}_1 = \mathcal{U}_{1.1} \bigcup \mathcal{U}_{1.2} :$$

$$\begin{pmatrix} \mathcal{U}_{1.2} = \{w_2,\ D_2\} \\ \mathcal{U}_{1.1} = \{w_1, D_1\} \end{pmatrix}, \begin{bmatrix} D_1 & D_2 \\ w_1 & w_2 \end{bmatrix} :$$

$$\begin{pmatrix} D_2 \\ \amalg \\ w_1 \end{pmatrix} \blacksquare \begin{pmatrix} w_2 \\ \amalg \\ D_1 \end{pmatrix} = \begin{pmatrix} D_2 & \circ & w_2 \\ \amalg & \diamond & \amalg \\ w_1 & \circ & D_1 \end{pmatrix}$$

**Bracket reading**

$$\begin{bmatrix} B_1 & B_2 \\ A_1 & A_2 \end{bmatrix}$$

*vertical* : " serial " *composition* $(\circ)$

*horizontal* : " parallel " mono –
contextural as *yuxtaposition* $(\bigotimes)$, poly – contextural as *mediation* $(\amalg)$.

**Generalized notation**

$$\mathcal{U}_1 = \mathcal{U}_{1.1} \bigcup \mathcal{U}_{1.2} :$$

$$\begin{pmatrix} \mathcal{U}_{1.2} = \{f_2,\ g_2\} \\ \mathcal{U}_{1.1} = \{f_1, g_1\} \end{pmatrix}, \begin{bmatrix} g_1 & g_2 \\ w_1 & w_2 \end{bmatrix} :$$

$$\begin{pmatrix} g_2 \\ \amalg \\ f_1 \end{pmatrix} \blacksquare \begin{pmatrix} f_2 \\ \amalg \\ g_1 \end{pmatrix} = \begin{pmatrix} g_2 & \circ & f_2 \\ \amalg & \diamond & \amalg \\ f_1 & \circ & g_1 \end{pmatrix}$$

$$w_1 \longrightarrow D_1 \ : \text{right – loop} = (w_1,\ D_1,\ D_2)$$

$$\updownarrow \ X \ \updownarrow \quad : \ X = \text{crosspoint}$$

$$D_2 \longrightarrow w_2 \ : \text{left – loop} = (w_2,\ D_1,\ D_2).$$

**Yuxtaposition: parallel (⊗)**
A mono-contextural modeling of the On-Off-mechanism, depending on two different temporal situations of the common domain, producing two sub-domains $\mathcal{U}_{1.2}$ and $\mathcal{U}_{1.2}$, is possible too. Instead of a polycontextural mediation of the two disjunct domains, a yuxtaposition "⊗" is defined over the objects of the common domain $\mathcal{U}_1$.

It seems to be natural to suppose a unity for the two domains $\mathcal{U}_{1.2}$ and $\mathcal{U}_{1.2}$,i.e. $\mathcal{U}_1$. This is presupposed by the definition of the memristive behavior as a pinched-hysteresis loop. But in more complex constellation such a unity has to be created by the system and has to be differentiated from other pinched-hysteresis loops from neighbor systems. Epistemologically, it also has to be distinguished between an internal and an external description of the behaviors. Both ways of description have to be harmonized towards an acceptable final characterization of memristive behaviors. All behaviors being involved together in the creation of a complex memristive system.

$$\textbf{Yuxtaposition: parallel}\left(\bigotimes\right)$$

$$\mathcal{U}_1 = \mathcal{U}_{1.1} \bigcup \mathcal{U}_{1.2} :$$

$$\left[\begin{array}{l} \mathcal{U}_{1.2} = \left\{\mathrm{R_{on\ 1.2}},\ \mathrm{R_{off\ 1.2}}\right\} \\ \mathcal{U}_{1.1} = \left\{\mathrm{R_{on\ 1.1}},\ \mathrm{R_{off\ 1.1}}\right\} \end{array}\right], \left[\begin{array}{ll} \mathrm{R_{off\ 1.1}} & \mathrm{R_{off\ 1.\ 2}} \\ \mathrm{R_{on\ 1.1}} & \mathrm{R_{on\ 1.2}} \end{array}\right]:$$

$$\begin{pmatrix} \mathrm{R_{on\ 1.2}} \\ \otimes \\ \mathrm{R_{on\ 1.1}} \end{pmatrix} \circ \begin{pmatrix} \mathrm{R_{off\ 1.2}} \\ \otimes \\ \mathrm{R_{off\ 1.1}} \end{pmatrix} = \begin{pmatrix} \mathrm{R_{on\ 1.2}} & \circ & \mathrm{R_{off\ 1.2}} \\ & \bigotimes & \\ \mathrm{R_{on\ 1.1}} & \circ & \mathrm{R_{off\ 1.1}} \end{pmatrix}$$

$$\begin{array}{|l|}\hline
\textbf{Yuxtaposition: cross-interchange} (\diamond) \\
\mathcal{U}_1 = \mathcal{U}_{1.1} \bigcup \mathcal{U}_{1.2}: \\
\left[ \begin{matrix} \mathcal{U}_{1.2} = \{R_{on\ 1.2}, R_{off\ 1.2}\} \\ \mathcal{U}_{1.1} = \{R_{on\ 1.1}, R_{off\ 1.1}\} \end{matrix} \right], \left[ \begin{matrix} R_{off\ 1.1} & R_{off\ 1.2} \\ R_{on\ 1.1} & R_{on\ 1.2} \end{matrix} \right]: \\
\begin{pmatrix} R_{off\ 1.2} \\ \diamond \\ R_{on\ 1.1} \end{pmatrix} \circ \begin{pmatrix} R_{off\ 1.1} \\ \diamond \\ R_{on\ 1.2} \end{pmatrix} = \begin{pmatrix} R_{off\ 1.2} & \circ & R_{off\ 1.1} \\ & \diamond & \\ R_{on\ 1.1} & \circ & R_{on\ 1.2} \end{pmatrix} \\ \hline
\end{array}$$

Considering the *temporal* change between On and Off in a single domain as $On_{1.1} \longrightarrow Off_{1.1}$ and $Off_{1.2} \longrightarrow On_{1.2}$, a cross-interchange ($\diamond$) between On and Off might be a more concrete modeling. The cross-interchange operation might be defined in monoidal categories with *yuxtaposition* “$\otimes$” and *permutation* ”$\sigma$”. Thus, the cross-interchange operator "$\diamond$” is used in this mono-contextural context as an abbreviation for "($\otimes$, $\sigma$).”

### 3.1.4. Iterations of pinched-hysteresis loops

***Parallel iteration***

$$\begin{array}{|l|}\hline
\begin{pmatrix} h_1 & h_2 \\ g_1 & g_2 \\ f_1 & f_2 \end{pmatrix} \in \mathcal{U}: \\
\begin{pmatrix} (f_1 \circ_1 g_1 \circ_1 h_1) \\ \otimes \\ (f_2 \circ_2 g_2 \circ_2 h_2) \end{pmatrix} = \\
\begin{pmatrix} f_1 \\ \otimes \\ f_2 \end{pmatrix} \circ \begin{pmatrix} g_1 \\ \otimes \\ g_2 \end{pmatrix} \circ \begin{pmatrix} h_1 \\ \otimes \\ h_2 \end{pmatrix} \\
\otimes : \text{yuxtaposition} \\
\circ : \text{composition} \\ \hline
\end{array}$$

***Two partly-overlapping pinched-hysteresis loops***

$$\text{loop}_1 \in \left(\mathcal{U}_{1.1} \bigcup \mathcal{U}_{1.2}\right),$$

$$\mathcal{U}_{1.1} = \mathcal{U}_{1.1\times.1} \circ \mathcal{U}_{1.1\times.2}, \quad \mathcal{U}_{1.2} = \mathcal{U}_{1.2\times.1} \circ \mathcal{U}_{1.2\times.2}$$

$$\text{loop}_2 \in \left(\mathcal{U}_{1.3} \bigcup \mathcal{U}_{1.4}\right)$$

$$\mathcal{U}_{1.3} = \left(\mathcal{U}_{1.1\times.1} \bigcup \mathcal{U}_{1.2\times.1}\right)$$

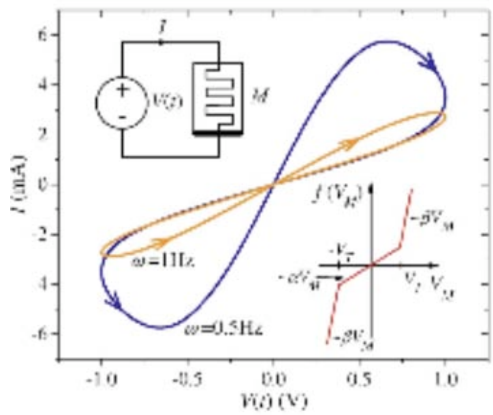
overlapping with loop $_1$

$$\mathcal{U}_1 = \mathcal{U}_{1.1} \bigcup \mathcal{U}_{1.2} \bigcup \mathcal{U}_{1.3} \bigcup \mathcal{U}_{1.4} :$$

$$\mathcal{U}_{1.i} = \left\{R_{\text{on } 1.i}, R_{\text{off } 1.i}\right\}, \; i = 1, 2, 3, 4 :$$

$$\left(\left(\begin{matrix} f_1 \\ \diamond_{1.2} \\ f_2 \end{matrix}\right) \circ_1 \circ_2 \left(\begin{matrix} g_1 \\ \diamond_{1.2} \\ g_2 \end{matrix}\right)\right) \circ_1 \circ_2 \left(\left(\begin{matrix} h_1 \\ \diamond_{1.2} \\ h_2 \end{matrix}\right) \circ_1 \circ_2 \left(\begin{matrix} k_1 \\ \diamond_{1.2} \\ k_2 \end{matrix}\right)\right) =$$

$$\left(\begin{matrix} (f_1 \circ_1 g_1) \circ_1 (h_1 \circ_1 k_1) \\ \diamond_{1.2} \\ (f_2 \circ_2 g_2) \circ_2 (h_2 \circ_2 k_2) \end{matrix}\right)$$




$$V(t) = V_o \sin(2\pi\omega t)$$

Rate of resistance change,

$$f(V_M) = -(\beta V_M + V_T(\alpha - \beta)\left[\left|V_M + V_T\right| - \left|V_M - V_T\right|\right])$$

### *Two pinched-hysteresis loops with common pinch*

h − loops : brown − loop $\in \mathcal{U}_{1.1}$, blue − loop $\in \mathcal{U}_{1.2}$

$$\mathcal{U}_1 = \mathcal{U}_{1.1\times.1} \bigcup \mathcal{U}_{1.1\times.2} \bigcup \mathcal{U}_{1.2\times.2}$$

$$\mathtt{h-loop}_1 \in \mathcal{U}_{1.1\times.1} \bigcup \mathcal{U}_{1.1\times.2},\ \mathtt{h-loop}_2 \in \mathcal{U}_{1.2\,.1} \bigcup \mathcal{U}_{1.2\times.2}$$

$$\mathtt{pinch}\left(\mathtt{h-loop}_1,\ \mathtt{h-loop}_2\right) = \mathtt{pinch}\left(\mathtt{h-loop}_1\right) \bigcap \mathtt{pinch}\left(\mathtt{h-loop}_2\right)$$

$$\mathcal{U}_1 = \mathcal{U}_{1.1\times.1} \bigcup \mathcal{U}_{1.1\times.2} \bigcup \mathcal{U}_{1.2\times.1} \bigcup \mathcal{U}_{1.2\times.2} :$$

$$\mathcal{U}_{1.2\times.2} = \left\{\mathrm{R_{on}}\ \mathtt{1.2\times.2},\ \mathrm{R_{off}}\ \mathtt{1.2\times.2}\right\}$$

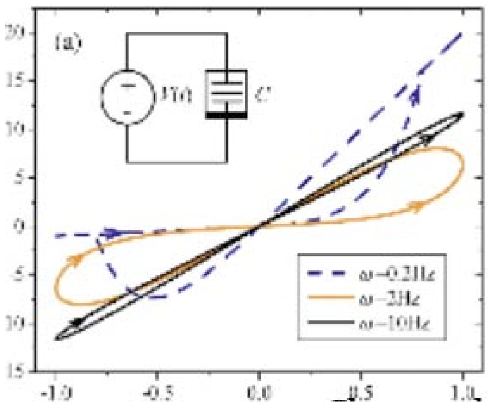
$$\left[\begin{array}{l} \mathcal{U}_{1.2\times.1} = \left\{\mathrm{R_{on}}\ \mathtt{1.2\times.1},\ \mathrm{R_{off}}\ \mathtt{1.2\times.1}\right\} \\ \mathcal{U}_{1.1\times.2} = \left\{\mathrm{R_{on}}\ \mathtt{1.1\times.2},\ \mathrm{R_{off}}\ \mathtt{1.1\,.2}\right\} \end{array}\right] :$$

$$\mathcal{U}_{1.1\times.1} = \left\{\mathrm{R_{on}}\ \mathtt{1.1\times.1},\ \mathrm{R_{off}}\ \mathtt{1.1\times.1}\right\}$$

$$\left(\begin{pmatrix} \mathrm{R_{on}}\ 1.2 \\ \diamond\ 1.2 \\ \mathrm{R_{on}}\ 1.1 \end{pmatrix} \circ \begin{pmatrix} \mathrm{R_{off}}\ 1.2 \\ \diamond\ 1.2 \\ \mathrm{R_{off}}\ 1.1 \end{pmatrix}\right)_1 \left[\diamond \circ\right] \left(\begin{pmatrix} \mathrm{R_{on}}\ 1.2 \\ \diamond\ 1.2 \\ \mathrm{R_{on}}\ 1.1 \end{pmatrix} \circ \begin{pmatrix} \mathrm{R_{off}}\ 1.2 \\ \diamond\ 1.2 \\ \mathrm{R_{off}}\ 1.1 \end{pmatrix}\right)_2 =$$

$$\begin{array}{c} \left(\mathrm{R_{on}}\ 1.2\times.1 \circ \mathrm{R_{off}}\ 1.2\times.1\right) \circ \left(\mathrm{R_{on}}\ 1.2\times.1 \circ \mathrm{R_{off}}\ 1.2\times.2\right) \\ \left[\diamond\right] \\ \left(\mathrm{R_{on}}\ 1.2\times.1 \circ \mathrm{R_{off}}\ 1.2\times.1\right) \circ \left(\mathrm{R_{on}}\ 1.2\times.1 \circ \mathrm{R_{off}}\ 1.2\times.2\right) \end{array}$$

## ***Three pinched-hysteresis loops***



$$\text{brown}-\text{loop} \in \mathcal{U}_{1.1} = \left(r-\text{loop}_1,\ l-\text{loop}_1\right)$$

$$\text{black}-\text{loop} \in \mathcal{U}_{1.2} = \left(r-\text{loop}_2,\ l-\text{loop}_2\right)$$

$$\text{blue}-\text{loop} \in \mathcal{U}_{1.3} = \left(r-\text{loop}_3,\ l-\text{loop}_3\right)$$

$$\mathcal{U}_1 = \mathcal{U}_{1.1} \bigcup \mathcal{U}_{1.2} \bigcup \mathcal{U}_{1.3} :$$

$$\begin{pmatrix} l-\text{loop}_1 \\ \diamond_{1.0\times.0} \\ r-\text{loop}_1 \end{pmatrix} \circ_{1.2\times.0} \begin{pmatrix} l-\text{loop}_2 \\ \diamond_{0.2\times.0} \\ r-\text{loop}_2 \end{pmatrix} \circ_{0.2\times.3} \begin{pmatrix} l-\text{loop}_3 \\ \diamond_{0.0\times.3} \\ r-\text{loop}_3 \end{pmatrix} =$$

$$\begin{pmatrix} \left(l-\text{loop}_1 \circ_{1.2\times.0} l-\text{loop}_2 \circ_{0.2\times.3} l-\text{loop}_3\right) \\ \diamond_{1.2\times.3} \\ \left(r-\text{loop}_2 \circ_{1.2\times.0} r-\text{loop}_2 \circ_{0.2\times.3} r-\text{loop}_2\right) \end{pmatrix}$$

## *Accretive modeling of tree pinched-hysteresis loops*

There might be a reason to thematize the three different pinched-hysteresis loops as being in some way incommensurable albeit their common and overdetermined pinch. In this case, a polyconcextural modeling with disjunct but mediated domains would be appropriate. All three loops meet at the same pinch, but this sameness is an overlapping or coincidence of their pinch and not an identity like in the previous modeling.

$$\begin{aligned}
\text{brown}-\text{loop} \in \mathcal{U}_1 &= \left(r-\text{loop}_1,\ l-\text{loop}_1\right)\\
\text{black}-\text{loop} \in \mathcal{U}_2 &= \left(r-\text{loop}_2,\ l-\text{loop}_2\right)\\
\text{blue}-\text{loop} \in \mathcal{U}_3 &= \left(r-\text{loop}_3,\ l-\text{loop}_3\right)
\end{aligned}$$

$$\mathcal{U}_1 \bigcap\nolimits_{1.2} \mathcal{U}_2 \bigcap\nolimits_{2.3} \mathcal{U}_3 = \emptyset$$

$$\mathcal{U}^{(3)} = \mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2 \amalg_{2.3} \mathcal{U}_3 :$$

$$\begin{pmatrix} \text{r}-\text{loop } 3 \\ \diamond_{0.2\times.3} \\ \text{r}-\text{loop}_2 \\ \diamond_{1.2\times.0} \\ \text{r}-\text{loop}_1 \end{pmatrix} \circ_{1.2\times.0} \begin{pmatrix} \text{l}-\text{loop}_3 \\ \diamond_{0.2\times.3} \\ \text{l}-\text{loop}_2 \\ \diamond_{1.2\times.0} \\ \text{l}-\text{loop}_1 \end{pmatrix} =$$

$$\begin{pmatrix} \left(r-\text{loop}_3 \circ_{0.0\times.3} l-\text{loop}_3\right) \\ \diamond_{0.2\times.3} \\ \left(\text{r}-\text{loop}_2 \circ_{0.2\times.0} l-\text{loop}_2\right) \\ \diamond_{1.2\times.0} \\ \left(\text{r}-\text{loop}_1 \circ_{1.0\times.0} l-\text{loop}_1\right) \end{pmatrix}$$

## 3.2. Distribution model

### 3.2.1. Distribution of the $R_{mem}$ – model

Based on the insight of the contextural locality of the occurrence of memristors a distribution and mediation of the basic concepts as they had been introduced by Stan Williams at the HP Lab, it might be opted for a dissemination of those constructs. First as a strict parallelism of the very same constructs for $R_{mem}$ at three discontextural loci.

$$\begin{pmatrix} R_{mem}(x)_1 \in (0, 1) & - \\ - & R_{mem}(x)_3 \in (0, 2) \\ R_{mem}(x)_2 \in (1, 2) & - \end{pmatrix}$$

$$x_i = \frac{w_i}{D_i} \in (0, 1)_i, \; i = 1, 2, 3 \,\Big|$$

----•----

↓ | ↓

–◇◇◇◇––•––◇◇◇◇– $R_{\text{mem1}} : (0, 1) = (0, 1)_1$

–◇◇◇◇––•––◇◇◇◇– $R_{\text{mem2}} : (1, 2) = (0, 1)_2$

–◇◇◇◇––•––◇◇◇◇– $R_{\text{mem3}} : (0, 2) = (0, 1)_3$

Memristors are depending on their material realizations. To conceptualize the behavior of memristive systems, the context in which such behavior is realized has to be considered too. Hence, memristors, logically modeled as *material implications* need a conceptual context to localize their logical distinctions.

In other words, it is not the case that the concept of material implication involved with memristors gets an implementation on a physical level as it happens for NAND-gates, which get a physical *inscription* into silicon on a microelectronic level, rather it is the nano-physical event of memristive behavior on nanoscale which enables a logical *interpretation* as a material implication. Again, *"the phenomena associated with memory are ubiquitous at the nanoscale"* (Di Ventra). This is not in contradiction to the fact that memristive devices too have to be artificially constructed.

Until now the modeling was focused on pinched-hysteresis loops as parallel or mediated loops with a *common* albeit not necessarily identical pinch-point. That is, the specification of the pinch-points with (0, 0) is not supposing a conceptual and physical identity but rather a kind of sameness, which allows an interactive mediation of the different loops.

A step further is introduced by the concept of *complementarity*, crucial for quantum physics.
Therefore, the next step of modeling will consider the poly-categorical approaches with distributed pinch-points.

### 3.2.2. Complementarity and disseminated pinched-loops

As it is mentioned before, time- and history -dependent events and features are ubiquitous on the nanosclae of quantum physics. What is also know quite well is that quantum physical events are not descriptable by classical logics. Especially the phenomenon of *complementarity* is demanding for a complex formalisms, i.e. logics, which are able to map complementarity. Hans Primas has developed those aspect with his concept of non-commensurable Boolean logics.

On the other hand it seems that memristic studies are neutral to such insights into complementarity. There is no mention about complementarity and no idea therefore, how complementarity and non-commensurability of different Boolean logics could have an influence on the definition of memristive systems.

The physical behavior of memristive devices is well described by *pinched-hysteresis loops*.
The question is: What happens with the pinched-hysteresis loops in a *complementary* setting?
Obviously, a complementarity demands for at least two different, not only disjunct but discontextural domains. In this sense, complementarity is different to duality. Therefore, the mono-contextural modeling in the frame of a single universe $\mathcal{U}_1$ with different domains $\mathcal{U}_{1.i}$, i∈N, has to be extended to a polycontextural modeling with mediated universes $\mathcal{U}^{(m)}$.

To elude the idea of *disseminated* "pinched-hysteresis loops" in a poly-contextural modeling, some citations from the eminent quantum theoretician Hans Primas might be of support by hinting to the background knowledge of a non-reductionist understanding of quantum theory.

**Complementarity**
*"Two Boolean descriptions are said to be complementary if they cannot be embedded into a single Boolean description."* (Primas, p. 17)

**Complementarity of tensed and tenseless time**
*"All known fundamental principles of physics refer to laws which are invariant under time translations. That is, the fundamental laws of physics do not contain any tensed notions. In contrast, one of the most the distinguishing qualities of consciousness is the Now."* (Primas, p. 29)

*"Since tensed and tenseless time refer to different domains, the still prevailing discussion whether the tensed or the tenseless theory of time is “true” makes no sense. Physical time is a crucial element in theoretical physics, but the experienced time cannot be dismissed as irrelevant for the understanding of physics. These two concepts of time are not contradictory but complementary. None of them is sufficient, none can replace the other, both are necessary.”* (Primas, p.33)

**Dressed electron**

*"The state representing an electron as actually observed in the laboratory is called a dressed electron,*
*it has a very complicated structure. Without an appropriate concept of an environmental background the concept of an individual quantum object makes no operational sense. Heuristically, a dressed electron can be thought of as consisting of the bare electron, interacting with its own radiation field by emitting and reabsorbing virtual photons. The presence of a virtual cloud does not only modify the properties of the bare elementary system and its bare environment, but also its dynamics. A dressed object is not only adapted to its environment. Its individuality emerges as by the interplay with the environment.”* (Primas, p. 33 )

### 3.2.3. Categorical descriptions of distributed interchangeability

**Parallel distributed interchangeability with distributed pinch – points**

$$\mathcal{U}_1 \bigcap\nolimits_{1.2} \mathcal{U}_2 = \emptyset$$

$$\mathcal{U}^{(2)} = \mathcal{U}_1 \coprod\nolimits_{1.2} \mathcal{U}_2 :$$

$$\begin{pmatrix} \mathcal{U}_2 = \{w_2,\ D_2\} \\ \mathcal{U}_1 = \{w_1,\ D_1\} \end{pmatrix}, \begin{bmatrix} D_1 & D_2 \\ w_1 & w_2 \end{bmatrix} :$$

$$\begin{pmatrix} w_2 \\ \coprod \\ w_1 \end{pmatrix} \circ \begin{pmatrix} D_2 \\ \coprod \\ D_1 \end{pmatrix} = \begin{pmatrix} w_2 & \circ & D_2 \\ & \coprod & \\ w_1 & \circ & D_1 \end{pmatrix}$$

$$\begin{pmatrix} w_2 & \circ & D_2 \\ & \coprod & \\ w_1 & \circ & D_1 \end{pmatrix} :: \begin{pmatrix} w_2 & \mid & D_2 \\ & \coprod & \\ w_1 & \mid & D_1 \end{pmatrix} = \begin{pmatrix} x_2 \\ \coprod \\ x_1 \end{pmatrix}$$

The interchangeability formula rules the *parallel* distribution of the w-D-distinction for two distributed memristors in poly-layered crossbar systems. The domains $\mathcal{U}_1$and $\mathcal{U}_2$**,** containing the different w-D-distinctions, might be defined by two distinctive ranges of TiO2 layers or two different physical domains. Thus, the two domains of the distributed memristors are physically and conceptually disjunct, $\mathcal{U}_1 \cap \mathcal{U}_1 = \emptyset$. The relationship of w and D, $\frac{w}{D}$, gets a categorization as a *composition* of morphisms, $w_2 \circ D_2$ and $w_1 \circ D_1$, and the distribution gets a categorization by the $\coprod$-functor for dissemination. As a first consequence of this mediated parallelism of memristive functionalities, one domain may represent digital events while the other may, simultaneously, represent analog events.

Because of the disjunctivity of the two domains $\mathcal{U}_1$ and $\mathcal{U}_2$, the objects of the categories are disjunct too. Therefore, the conditions for an interpretation of distribution as *yuxtaposition* in the sense of monoidal categories are not given. Monoidal categories are mono-contextural and the objects of composition and yuxtaposition belong to the same common domain (universe). Hence, monoidal composition and yuxtaposition are referring to the same universe of objects, and are therefore mono-contextural.

Considering the distribution of the $R_{on}$ – and $R_{off}$ –aspects of the two disjunct domains, the interchangeability takes the following formulation.

### 3.2.4. Super-additivity of categorical descriptions

**balanced 3 – contextural interchangeability with super – additivity**

$$\mathcal{U}_1 \bigcap\nolimits_{1.2} \mathcal{U}_2 \bigcap\nolimits_{2.3} \mathcal{U}_3 = \emptyset$$

$$\mathcal{U}^{(3)} = \mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2 \amalg_{2.3} \mathcal{U}_3 :$$

$$\mathcal{U}_i = \{w_i, D_i\}, \ i = 1, 2, 3$$

$$\begin{bmatrix} D_1 & D_2 & D_3 \\ w_1 & w_2 & w_3 \end{bmatrix}:$$

$$\left( \begin{array}{c} \left( \begin{array}{c} \left(w_1 \circ^{1.0\times.0} D_1\right) \\ \amalg_{1.2\times.0} \\ \left(w_2 \circ^{0.2\times.0} D_2\right) \end{array} \right) \\ \amalg_{1.2\times.3} \\ \left(w_3 \circ^{0.0\times.3} D_3\right) \end{array} \right) = \left( \begin{array}{c} \left( \begin{array}{c} w_1 \\ \amalg_{1.2\times.0} \\ w_2 \end{array} \right) \\ \amalg_{1.2\times.3} \\ w_3 \end{array} \right) \circ_{1.2\times.3} \left( \begin{array}{c} \left( \begin{array}{c} D_1 \\ \amalg_{1.2\times.0} \\ D_2 \end{array} \right) \\ \amalg_{1.2\times.3} \\ D_3 \end{array} \right)$$

The interchangeability formula rules the parallel distribution of the w-D-distinction for two memristors. The domains $\mathcal{U}_1, \mathcal{U}_2$ and $\mathcal{U}_3$ containing the different w-D-distinctions, might be defined by three distinctive ranges of TiO2 layers. The relationship of w and D, $\frac{w}{D}$, (w ı D), gets a categorization as a *composition* of morphisms, $w_3 \circ D_3$, $w_2 \circ D_2$ and $w_1 \circ D_1$.

$$
\left(\begin{array}{c}
\left(\begin{array}{c}
\left(w_1 \circ^{1.0\times.0} D_1\right) \\
\amalg_{1.2\times.0} \\
\left(w_2 \circ^{0.2\times.0} D_2\right)
\end{array}\right) \\
\\
\amalg_{1.2\times.3} \\
\left(w_3 \circ^{0.0\times.3} D_3\right)
\end{array}\right)
\Longrightarrow
\left(\begin{array}{c}
\left(\begin{array}{c}
\left(w_1 \mid^{1.0\times.0} D_1\right) \\
\amalg_{1.2\times.0} \\
\left(w_2 \mid^{0.2\times.0} D_2\right)
\end{array}\right) \\
\\
\amalg_{1.2\times.3} \\
\left(w_3 \mid^{0.0\times.3} D_3\right)
\end{array}\right)
=
\left(\begin{array}{c}
\left(\begin{array}{c}
\left(x_1\right) \\
\amalg_{1.2\times.0} \\
\left(x_2\right)
\end{array}\right) \\
\\
\amalg_{1.2\times.3} \\
\left(x_3\right)
\end{array}\right)
$$

This approach to modeling might be slightly too abstract but fits well into the conceptual modeling language and the concrete behaviors to be modeled. It also serves as steps towards a generalization of the concrete approaches for applications to other domains.

**Generalizations**

It follows naturally from the interchangeability formula, that it holds independently from specific domains. Hence, instead of voltage for nano-electronic devices, like memristors, capacity or inductivity might be involved, producing mem-capacitors or mem-inductors. But as Leon Chua made it clear, mem-behaviors appear in *memristive systems* under many different definitions and physical domains (universes).

It seems not to be too wrong to apply this kind of thinking to cognitive, mental, psychic, social and other domains too. This might be understood as a hint to analyze the different applications of second-order cybernetic constructions, like *autopoiesis* and *re-entry* under the focus of memristics. Obviously, such generalizations have not to be restricted to two and only two domains at all.

### 3.2.5. General cross-interchangeability

**Cross-interchangeability for 2-contextures**

$$\forall f_i, g_i \in \text{Universe } \mathcal{U}_i, i = 1, 2$$
$$\forall i \neq j: \mathcal{U}_i \bigcap \mathcal{U}_j = \emptyset$$

$$\begin{pmatrix} \mathcal{U}_2 \\ \mathcal{U}_1 \end{pmatrix}, \begin{pmatrix} g_1 & g_2 \\ f_1 & f_2 \end{pmatrix}:$$

$$\begin{pmatrix} g_2 \\ \amalg \\ f_1 \end{pmatrix} \diamond \begin{pmatrix} f_2 \\ \amalg \\ g_1 \end{pmatrix} = \begin{pmatrix} g_2 & \circ & f_2 \\ & \blacksquare & \\ f_1 & \circ & g_1 \end{pmatrix}$$

$\amalg$ : mediation between contextures

$\circ$ : composition of morphisms

$\diamond$ : cross – interchange between levels

$\blacksquare$ : $(\diamond, \amalg)$, mediated cross – interchange

$=$ : equivalence

The topics studied now on this level of abstraction are the *loops* and their *pinch*-points. Hence, the distribution and mediation of full loops and their corresponding pinches has to be thematized. Before, the internal structure of the loops as left- and right-loops with pinch-point had been in focus.
Hence, what has to be brought into interaction now, are the distributed *loops* and the distributed *pinches* of different loci. A model to think about such a dissemination is intended by the concept to be developed of *poly-layered* crossbar systems.
http://www.thinkartlab.com/Memristics/Poly-Crossbars/Poly-Crossbars.pdf

**Orientation**

The aim of such tedious formalisms, like the poly-categorical interchangeability formulas, is to give *orientation* to understand the interplay of different kinds of memristive devices in a complex constellation in the framework of a theory of living systems.

**Mutual cross – exchange of loops and pinches**

$$\mathcal{U}_i = \left\{ \text{loop}_i, \text{pinch}_i \right\}, \ i = 1, 2$$

$$\mathcal{U}_1 \bigcap\nolimits_{1.2} \mathcal{U}_2 = \emptyset$$

$$\mathcal{U}^{(2)} = \mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2 :$$

$$\begin{bmatrix} \text{pinch}_1 & \text{pinch}_2 \\ \text{loop}_1 & \text{loop}_2 \end{bmatrix} :$$

$$\begin{pmatrix} \text{pinch}_2 \\ \amalg \\ \text{loop}_1 \end{pmatrix} \diamond \begin{pmatrix} \text{loop}_2 \\ \amalg \\ \text{pinch}_1 \end{pmatrix} = \begin{pmatrix} \text{pinch}_2 & \circ & \text{loop}_2 \\ & \blacksquare & \\ \text{loop}_1 & \circ & \text{pinch}_1 \end{pmatrix}$$

**Mutual cross – exchange of loops and pinches**

$$\mathcal{U}_i = \{\text{loop}_i, \text{pinch}_i\},\ i = 1, 2, 3$$

$$\mathcal{U}_1 \bigcap_{1.2} \mathcal{U}_2 \bigcap_{2.3} \mathcal{U}_3 = \emptyset$$

$$\mathcal{U}^{(3)} = \mathcal{U}_1 \amalg_{1.2} \mathcal{U}_2 \amalg_{2.3} \mathcal{U}_3 :$$

$$\begin{bmatrix} \text{pinch}_1 & \text{pinch}_2 & \text{pinch}_3 \\ \text{loop}_1 & \text{loop}_2 & \text{loop}_3 \end{bmatrix}$$

$$\left( \begin{array}{c} \left( \begin{array}{c} \left(\text{loop}_1 \circ^{1.0\times.0} \text{pinch}_1\right) \\ \blacksquare_{1.2\times.0} \\ \left(\text{pinch}_2 \circ^{0.2\times.0} \text{loop}_2\right) \end{array} \right) \\ \amalg_{1.2\times.3} \\ \left(\text{loop}_3 \circ^{0.0\times.3} \text{pinch}_3\right) \end{array} \right) =$$

$$\left( \begin{array}{c} \left( \begin{array}{c} \text{loop}_1 \\ \amalg_{1.2\times.0} \\ \text{pinch}_2 \end{array} \right) \\ \amalg_{1.2\times.3} \\ \text{loop}_3 \end{array} \right) \begin{bmatrix} \diamond_{1.2\times.0} \\ \circ_{0.0\times.3} \end{bmatrix} \left( \begin{array}{c} \left( \begin{array}{c} \text{pinch}_1 \\ \amalg_{1.2\times.0} \\ \text{loop}_2 \end{array} \right) \\ \amalg_{1.2\times.3} \\ \text{pinch}_3 \end{array} \right)$$

### 3.2.6. Diamond interchangeability

Diamond interchangeability enters the game if the fact of the importance of environments of operations are taken seriously and modeled in a specific formalism, like the proposed diamond category theory.

**In-sourcing as memorization**

In another turn of the argumentation for diamond-theoretic combinations, i.e. composition and saltisition, the role of memory as the "in-sourcing" of the matching conditions of compositions, which are inscribed as saltisitions, might be emphasized. It might become clear that this mechanism of in-sourcing is possible only if the mutual interaction of memory as saltisition and computation as composition is considered. Therefore, diamond combination might be realized in a memristive system as an interaction of memory and computation. Diamond systems are physically realizable as an interaction of memristors as memory and memristors as computation. Or in another setting, as an interaction between memristors and classical emulations, like NAND-gates.

The complementary construction proposed in another paper was to 'deduce' memristance from diamond-categorical properties. This is a *conceptual* approach to memristive systems. The inverse move, to realize diamond-behaviors within the framework of memristive system is the *physical* approach to a transclassical understanding of technology.

The *complementarity* of conceptual and physical approaches corresponds the complementarity which Hans Primas is stressing in his research: the complementarity of Mind and Matter. Quantum physics isn't possible without an understanding of its thematization, i.e. quantum physics is involving observer-positions to be observed (Exo-/Endophysics).

Hence, a further understanding of diamond category theory is possible with the concept and physics of memristive devices. To realize the in-sourcing of the matching conditions while a composition on the base of its matching conditions happens implies a remembrance of it. This remembrance is inscribed in the realm of saltisitions. Again, the interplay between composition and saltisition might be understood as an interaction between computation and memorization.